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Robust $\mathcal{H}_2$ Performance for Sampled-data Systems

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Abstract

Robust $\mathcal{H}_2$ performance conditions under structured uncertainty, analogous to well known methods for $\mathcal{H}_\infty$ performance, have recently emerged in both discrete and continuous-time. This paper considers the extension into uncertain sampled-data (SD) systems, taking into account inter-sample behavior. Convex conditions for robust $\mathcal{H}_2$ performance are derived for different uncertainty sets.

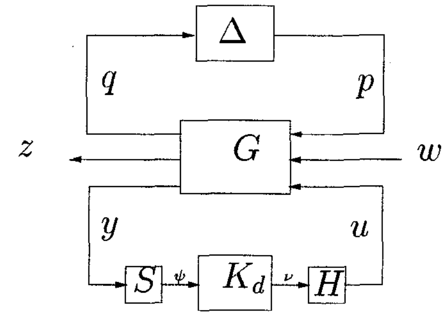

Figure 1: Uncertain Sampled-data System

1 Introduction & Background

In Dullerud [Dul95], a thorough study of robustness analysis for sampled-data systems was undertaken. In the configuration of Figure 1, $\boldsymbol{G}$ is a continuous time linear time-invariant (LTI) system, controlled by a discrete-time LTI controller $\mathbf{K_d}$ by means of ideal sample and hold devices with synchronized period h. This makes the nominal closed loop map $\boldsymbol{M}$ (from (p, w) to (q, z)) periodically time varying (PTV), instead of LTI as is usual in robust control. The system is affected by dynamic uncertainty Δ, which has spatial structure and can be LTI, PTV, or arbitrarily time-varying (LTV). Methods for robust stability and $\mathcal{H}_\infty$ performance evaluation were studied in [Dul95], extending the standard theory for continuous or discrete time systems.

In this paper we consider the question of robust $\mathcal{H}_2$ performance for sampled data systems, following recent results in [Pag96a, Pag96b] in the standard case, which closely resemble the robust $\mathcal{H}_\infty$ theory. For sampled data systems, we extend these conditions for both PTV and LTV perturbations.

1.1 Lifting

For a general introduction see [CF95] and references therein. The Laplace, Lift and the Λ (or Z) transforms between the signal spaces are related as seen below, we use the same accents (from [Dul95]) for operators mapping within the domains.

$$\begin{tikzcd} \check{f}(\lambda) & \tilde{f}(k) \in \boldsymbol{\ell_2} \triangleq \ell_{\mathcal{L}_2[0;h[} \arrow[l, "\Lambda"'] \\ \hat{f}(s) \arrow[u, "\Lambda L \mathcal{L}^{-1}"] & f(t) \in \mathcal{L}_2 \arrow[l, "\mathcal{L}"] \arrow[u, "L"'] \end{tikzcd} \tag{1}$$

The lifting technique converts the PTV operator $\boldsymbol{M}$ to LTI in the lifted domain. In the Λ-domain, it amounts to the operator $\check{f}(\lambda) \mapsto \check{M}(\lambda)\check{f}(\lambda)$, where at each λ in the unit disk, $\check{M}(\lambda)$ is an operator on $\mathcal{L}_2[0; h[$.

1.2 $\mathcal{H}_2$ Performance for TV systems

For LTI systems the $\mathcal{H}_2$ norm is given by

$$\|\boldsymbol{T}\|_2^2 \triangleq \frac{1}{2\pi} \int_{-\infty}^{\infty} \operatorname{trace}(\hat{T}(j\omega)^* \hat{T}(j\omega)) d\omega = \sum_{i=1}^{n} \|\boldsymbol{T}\delta e_i\|_{\mathcal{L}_2}^2$$

where $\boldsymbol{T}\delta e_i$ is the impulse response for the i-th input channel. For TV systems, this impulse response varies in time; one possible generalization of the $\mathcal{H}_2$ norm used in [BJ92] (a different one is given in [Pag96b]) is to average over time. For PTV systems, we average over the period:

$$\|\boldsymbol{T}\|_{\mathcal{H}_2}^2 = \frac{1}{h} \int_0^h \sum_{i=1}^{n} \|\boldsymbol{T}\delta_\tau e_i\|_{\mathcal{L}_2}^2 d\tau \tag{2}$$

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Going to the Λ domain, this is equivalent to the form given in (3), where $||\cdot||_{\mathbb{HS}}$ is the Hilbert-Schmidt norm of an operator on $\mathcal{L}_2[0;h[$, see [BJ92, CF95].

$$||\check{T}||^2_{\mathcal{H}_2} = \frac{1}{2\pi}\int_0^{2\pi} ||\check{T}(e^{j\theta})||^2_{\mathbb{HS}} d\theta \tag{3}$$

For arbitrary LTV systems (2) may be generalized by taking the limit as $h \to \infty$.

2 Robust $\mathcal{H}_2$ Performance SD for TV uncertainty

The sets of full block structured LTV and PTV perturbations (of period h) are given by

$$\boldsymbol{\Delta}_{LTV} \triangleq \{\boldsymbol{\Delta} = \mathrm{diag}(\boldsymbol{\Delta}_1, ..., \boldsymbol{\Delta}_F) : \boldsymbol{\Delta}_k \in \mathfrak{L}(\mathcal{L}_2^{m_k})\}$$
$$\boldsymbol{\Delta}_{PTV} \triangleq \{\boldsymbol{\Delta} \in \boldsymbol{\Delta}_{LTV} : D_h\boldsymbol{\Delta} = \boldsymbol{\Delta}D_h\}$$

$\boldsymbol{T}_{zw}(\boldsymbol{\Delta})$ denotes the map from w to z (see Fig. 1).

2.1 PTV perturbation case

Let $\Delta \in \boldsymbol{\Delta}_{PTV}$. At each $\lambda = e^{j\theta}$, we introduce a scaling which commutes with $\check{\Delta}(e^{j\theta})$, $X(\theta) \in \mathbb{X} \triangleq \{X = \mathrm{diag}[x_1 I_{m_1}, ..., x_F I_{m_F}], x_k > 0\}$ (constant matrix multiplication operator on $\mathfrak{L}(\mathcal{L}_2[0;h[)$).

Condition 1 *There exists functions $X(\theta) \in \mathbb{X}$ and $Y(\theta) \in \mathfrak{L}(\mathcal{L}_2[0;h[)$, such that*

$$\check{M}(e^{j\theta})^* \begin{bmatrix} X(\theta) & 0 \\ 0 & I \end{bmatrix} \check{M}(e^{j\theta}) - \begin{bmatrix} X(\theta) & 0 \\ 0 & Y(\theta) \end{bmatrix} < 0 \tag{4}$$

for all $\theta \in [0;2\pi[$ and

$$\int_0^{2\pi} \mathrm{tr}\, Y(\theta)\frac{d\theta}{2\pi} = \int_0^{2\pi}\int_0^h \mathrm{trace}\, Y_\theta(t,t)dt\frac{d\theta}{2\pi} < 1. \tag{5}$$

Remark 1 *In (5) we use the* trace *of an operator $Y \in \mathfrak{L}(\mathcal{L}_2[0;h[)$; this is defined as*

$$\mathrm{tr}\, Y \triangleq \sum_{i=1}^{\infty}\langle Yb_i, b_i\rangle = \int_0^h \mathrm{trace}\, Y(t,t)dt, \tag{6}$$

where b_i is any orthonormal basis of $\mathcal{L}_2[0;h[$, and $Y(t,\tau)$ is the kernel representation of Y.

Proposition 1 *If Condition 1 holds and $\boldsymbol{\Delta} \in \boldsymbol{B}_{\boldsymbol{\Delta}_{PTV}}$, then the system is robustly stable and*

$$\sup_{\boldsymbol{\Delta}\in \boldsymbol{B}_{\boldsymbol{\Delta}_{PTV}}} ||\boldsymbol{T}_{zw}(\boldsymbol{\Delta})||_{\mathcal{H}_2} < 1. \tag{7}$$

Proof: See [RP97].

Remark 2 *This sufficient condition is convex in the unknowns $X(\theta)$, $Y(\theta)$. The "frequency" and "time" dependence of $\check{M}$ is reflected in $Y_\theta(t,t) \in \mathbb{C}^{m\times m}$. A finite dimensional approximation can be obtained by gridding. Clearly, this condition also holds for LTI perturbations, however, the LTI behaviour can be further exploited (see [RP97]).*

2.2 LTV perturbation case

Proposition 2 *If Condition 1 holds for a constant function $X(\theta) \equiv X \in \mathbb{X}$, and $\boldsymbol{\Delta} \in \boldsymbol{B}_{\boldsymbol{\Delta}_{LTV}}$, then the uncertain system is robustly stable and*

$$\sup_{\boldsymbol{\Delta}\in \boldsymbol{B}_{\boldsymbol{\Delta}_{LTV}}} ||\boldsymbol{T}_{zw}(\boldsymbol{\Delta})||_{\mathcal{H}_2} < 1. \tag{8}$$

3 Conclusion and further directions

Conditions for robust $\mathcal{H}_2$ performance for sampled-data systems have been derived under time-varying uncertainty (PTV or arbitrary LTV). Only sufficiency was shown; it is expected that necessity results will follow if one adopts the notion of $\mathcal{H}_2$ performance in [Pag96a, Pag96b], and replaces PTV uncertainty by a "quasi-PTV" notion (see [Dul95]). Further work includes state-space computations for these conditions, and more refined conditions for the case of purely LTI uncertainty.

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